The University of Alabama at Birmingham (UAB)
Department of Physics

PH 561 – Classical Mechanics I – Fall 2005

Integrative Project Assignment

(1) Solve the attached list of problems. **Due Tuesday, November 22, 2005**
(2) In consultation with instructor select *one* problem whose solution will be delivered as an oral presentation to the class on **Thursday, December 1, 2005.**

List of Problems (Due, November 22, 2005)

1. A particle of mass m with initial speed v_0 at $t = -\infty$ is acted upon by a time dependent impulsive force given by:

$$F(t) = \frac{I_0 \delta t}{\pi} \frac{1}{(t - t_0)^2 + (\delta t)^2}; \qquad \text{for} \quad -\infty < t < \infty$$

   where δt is a very small time interval compared to the total travel time of the particle.
   a. Graph the force $F(t)$ indicating explicitly on your graph the meaning of the various parameters that appear on the expression above.
   b. Calculate the total impulse delivered by $F(t)$ to the particle.
   c. What is the final speed of the particle at $t \to \infty$?
   d. Apply this model force, $F(t)$, to the case of an elastic collision of the particle with a rigid wall. Assuming $m = 0.1$ kg, $v_0 = 10$ m/s, and a collision time $\delta t = 0.001$ s, estimate the magnitude of the force experienced by the particle due to the impact. Compare the value you found with the weight of the particle.

2. A particle of mass m moves in one dimension under the effect of a force whose potential energy is given by:

$$V(x) = -\tfrac{a}{3} x^3 + \tfrac{b}{2} x^2$$

   a. Graph this potential energy and discuss the various types of motion that may take place depending on the total energy of the system.

   b. Draw the corresponding phase space trajectories for various values of the total energy and analyze how the details of the trajectories relate to the graph of the potential energy.

3. A particle of mass m falls vertically through a fluid under a resistive force $-b_2v^2$ where b_2 is a positive constant and v is the particle velocity. Find the motion of the particle and compare it with the case of linear resistive force.

4. Find the motion of an underdamped oscillator $[\gamma = (1/3)\omega_0]$ initially at rest and subject, after t =0, to a force

$$F = A \sin \omega_0 t + B \sin 3\omega_0 t,$$

where ω_0 is the natural frequency of the oscillator.

What ratio B to A is required in order for the forced oscillation at frequency $3\omega_0$ to have the same amplitude as that at frequency ω_0?

Suggestion: Use the principle of superposition discussed in section 3.9 of the textbook (Fowles & Cassiday, 7th Edition).

5. A particle of mass m moves in a plane subjected to a force that may be expressed as follows:

$$\mathbf{F} = -k\mathbf{r}$$

where k is a constant and $\mathbf{r}$ is the position vector of the particle with respect to the origin.

a. What can you say about the total mechanical energy of this particle? (Show explicitly how you support your answer).

b. Determine the equations of motion of the particle in the Cartesian coordinate system.

c. Determine the trajectory of the particle if it is set in motion with the following initial conditions:

$$\mathbf{r_0} = \mathbf{i}x_0 \text{ (i.e., } x_0 \text{ along the } x\text{-axis)}$$
$$\mathbf{v_0} = \mathbf{j}v_0 \text{ (i.e., } v_0 \text{ along the } y\text{-axis)}$$

The particle is now given an electrostatic charge q and a weak uniform magnetic field $\mathbf{B}=B_0\mathbf{k}$ (i.e., perpendicular to the plane of motion) is applied to the system.

d. Write the new equations of motion for the particle.

e. Find and discuss the new trajectory of the particle once the magnetic field has been turned on.